Packing A-paths in Group-Labeled Graphs via Linear Matroid Parity

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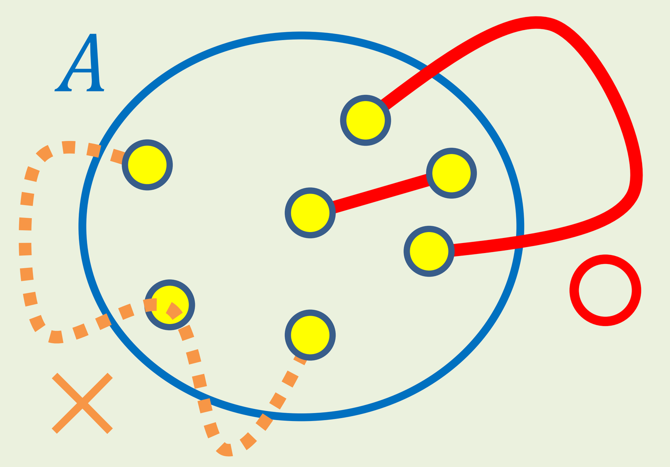
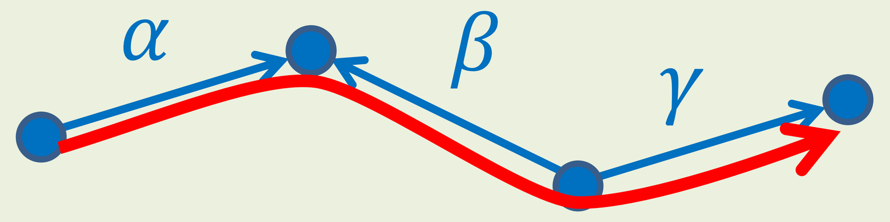
1. Packing <u>A-paths</u> in <u>Group-Labeled Graphs</u>

Paths **between** $A \subseteq V$ **NOT through** A



$G = (V, E)$: directed graph
$\psi: E \to \Gamma$ (Γ: group)

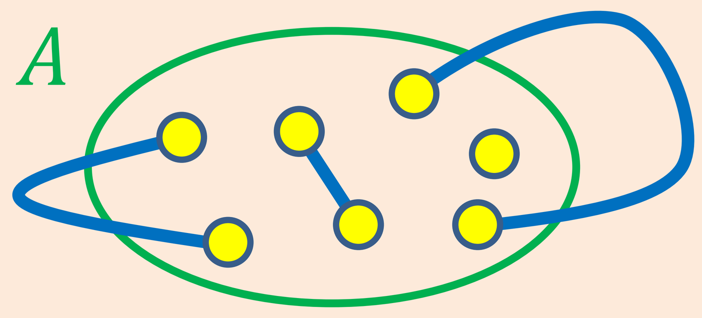


W: (undirected) walk
$\psi(W) \coloneqq \gamma \cdot \beta^{-1} \cdot \alpha$

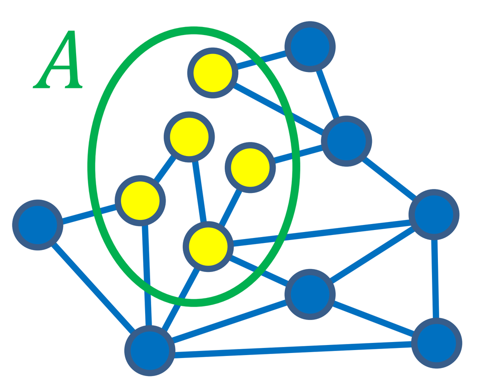
[Problem 1] <u>Subgroup Model</u>

Input: (G, ψ): Γ-labeled graph
$A \subseteq V(G)$: terminal set
Γ': proper **<u>subgroup</u>** of Γ

Find: a maximum family of vertex-disjoint **<u>admissible</u>** A-paths



2. Reduction to Linear Matroid Parity (L.M.P.)

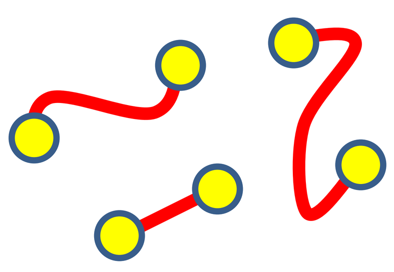


edge ↕ column-pair

feasible edge set ↕ feasible pairs

<u>unique</u> extraction

vertex-disjoint **<u>admissible</u>** A-paths



[Problem 2] <u>Linear Matroid Parity</u>

Input: a matrix in $\mathbf{F}^{n \times 2m}$ ($\mathbf{F}$: field) with **<u>pairing of the columns</u>**

Find: a maximum family of pairs with **<u>the linear independence</u>**

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

3. <u>Main Theorem</u>

Γ: group, Γ': subgroup of Γ, $\mathbf{F}$: field

$^\exists \rho: \Gamma \to \mathrm{PGL}(2, \mathbf{F})$ homomorphic
$^\exists Y$: 1-dimentional subspace of $\mathbf{F}^2$
s.t. $\Gamma' = \{\, \alpha \in \Gamma \mid \rho(\alpha) Y = Y \,\}$

⇕

Subgroup model reduces to L.M.P. with **<u>coherent representation</u>** over $\mathbf{F}$.

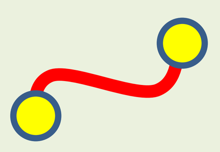
(i) Based on **<u>incidence matrix</u>**.

$e = uv \in E$

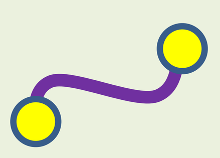
$$\begin{matrix} \\ u \\ \\ v \\ \\ \end{matrix} \begin{bmatrix} O \\ * \\ O \\ * \\ O \end{bmatrix} \quad * \in \mathbf{F}^{2 \times 2}$$

paired

(ii) Correspondence of **<u>feasibility</u>**.



admissible → feasible in L.M.P.

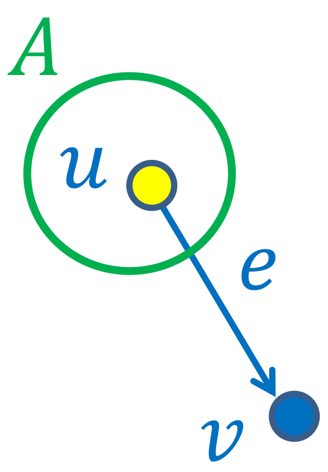


non-admissible → infeasible in L.M.P.

<u>How to Construct a Matrix</u>

$\mathrm{GL}(n, \mathbf{F})$: the set of nonsingular $n \times n$ matrices over $\mathbf{F}$
$\mathrm{PGL}(n, \mathbf{F}) \coloneqq \mathrm{GL}(n, \mathbf{F}) / \{\, kI_n \mid k \in \mathbf{F} \,\}$

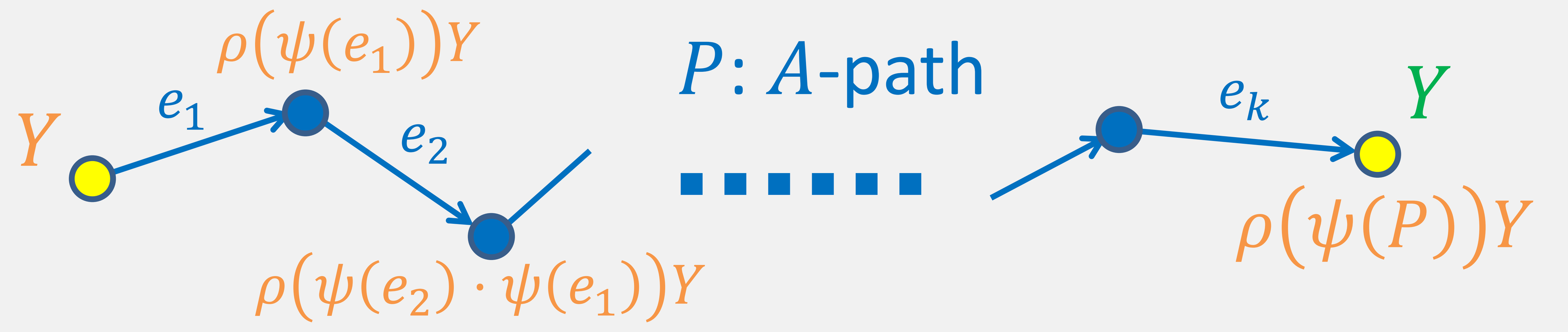
ρ: **projective representation** of Γ with Y **fixed** w.r.t. Γ'



	e	$\mathbf{0} \neq y \in Y$
	O	$\mathbf{0}$
u	I_2	y
	O	
v	$-\rho(\psi(e))$	$\mathbf{0}$
	O	

Eliminate in advance. ⇔ **Begin with these.**

<u>Observation</u>



$\psi(P) \in \Gamma' \iff \rho(\psi(P))Y = Y$